Exercise 1

Find the principal argument Arg z when

$$(a) \ \ z = \frac{i}{-2 - 2i}; \qquad (b) \ \ z = \left(\sqrt{3} - i\right)^6.$$

$$Ans. \ \ (a) \ \ -3\pi/4; \quad (b) \ \ \pi.$$

Solution

Part (a)

$$\arg z = \arg\left(\frac{i}{-2 - 2i}\right)$$

$$= \arg(i) - \arg(-2 - 2i)$$

$$= \left(\frac{\pi}{2}\right) - \left(\tan^{-1}\frac{-2}{-2} + \pi\right) + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$= \frac{\pi}{2} - \left(\frac{\pi}{4} + \pi\right) + 2n\pi$$

$$= -\frac{3\pi}{4} + 2n\pi$$

Since the principal argument Arg z is required to be between $-\pi$ and π ($-\pi < \text{Arg } z \leq \pi$), we choose $n = 0$.

$$\text{Arg } z = -\frac{3\pi}{4}$$

Part (b)

Switch to polar form first to deal with the exponent.

$$\arg z = \arg\left(\sqrt{3} - i\right)^6$$

$$= \arg\left[\sqrt{(\sqrt{3})^2 + (-1)^2}\exp\left(i\tan^{-1}\frac{-1}{\sqrt{3}}\right)\right]^6$$

$$= \arg\left[\sqrt{4}\exp\left(-i\frac{\pi}{6}\right)\right]^6$$

$$= \arg\left(2e^{-i\pi/6}\right)^6$$

$$= \arg\left(64e^{-i\pi}\right)$$

$$= -\pi + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

Since the principal argument Arg z is required to be between $-\pi$ and π ($-\pi < \text{Arg } z \leq \pi$), we choose $n = 1$.

$$\text{Arg } z = \pi$$